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Formation Control of Swarm Quadrotors

Amir Noabahar Sadeghi Nam

Abstract - This paper presents design of control systems for a group of quadrotors performing formation flight. The system has been presented into two interconnected subsystems. The first one representing the under-actuated subsystem gives the dynamic relation of the altitude and attitude. The second fully-actuated subsystem gives the dynamics of the horizontal position. To design motion control system, PID controller, active disturbance rejection (ADRC) controller and back stepping control architecture are designed and presented. Extended state observer (ESO) technique has been studied in order to eliminate the disturbances. The performances measure of the control architectures are computed to compare the presented control architectures more accurately.

Index Terms — Swarm Quadrotors, Active Disturbance Rejection Control, Extended State Observer, Back Stepping Control

1 Introduction

A quadrotor or quadcopter is a rotary wing unmanned aerial vehicle with four rotors placed on the ends of a two bars cross structure. Its flight operation is controlled by varying the rotational speeds of each rotor. The quadrotor swarms are multiple quadrotors which are employed to accomplish a common task.

Swarm quadrotors has took an important part of the recent researches. For recent surveys on this subject, some techniques such as feedback linearization, nonlinear optimal control, back stepping control , sliding mode control and classic PID have been presented to achieve this application. Study [1], investigates the formation control problems for quadrotor swarm systems. A quadrotor is modeled dynamically as a point-mass system by double integrator. To achieve the desired time-varying formation, a consensus based formation architecture is presented. A new scheme for trajectory tracking of swarm quadrotors under a centralized leader-followers formation strategy is proposed in the article [2]. First, a double loop control structure based on the linear quadratic regulator is presented to control the horizontal position and stabilize the attitude. Then sliding mode control approach is employed to leader-followers formation. Feedback linearization with dynamic extension has been used to develop of a nonlinear position controller for a quadrotor aircraft in [3]. In another survey [4], a nonlinear optimal control scheme has been proposed to autonomous flight of quadrotors. The control system consists of a nonlinear model predictive controller and a nonlinear disturbance observer. Integral back stepping control approach has been described to full control of a quadrotor in [5]. The author in [6] have designed and presented a controller based on the classic scheme of PID control, which aims to regulate the posture of a six degree-of-freedom quadrotor.

The other main issue in this regard is the uncertainties, which are universal in practice and of course in our system. They usually originate from two sources: internal (parameter or structure) uncertainty and external (disturbance) uncertainty. Lots of control methods have been proposed in literature centering on this issue, such as the widely used PID control, adaptive control, robust control, etc. What's more, many disturbance estimating techniques appeared, such as disturbance observer (DOB), active disturbance rejection controller (ADRC), extended state observer (ESO) and etc. The recently developed sliding mode control driven by sliding mode disturbance observer approach is used to design a robust flight controller for a small quadrotor vehicle in [7]. Ref [8] employs a nonlinear disturbance observer to design a robust trajectory tracking controller for quadrotors. Finally an extended observer is designed and presented in [9] to estimate the disturbance in order to attitude control of a quadrotor aircraft.

In this paper, designing of flight control systems for swarm quadrotors are studied. The quadrotor dynamics is presented into two interconnected subsystems. The first under-actuated subsystem gives the dynamics of the vertical position with the yaw, pitch and roll angles, and the second fully-actuated subsystem gives the dynamics of the horizontal position. To design motion control system, two methodologies are presented: classic PID controller and active disturbance rejection controller (ADRC) along with back stepping control architecture. Ability of extended state observer (ESO) to eliminate the external disturbances technique are evaluated. To compare the behavior of both methodologies, the performances measure of the control architectures are extracted. These techniques and contributions, reveal novelties for the control system design of swarm quadrotors in the theoretical domain.

2 Quadrotor Dynamic Model

The dynamic model of a quadrotor represents its motion mathematically in some equations of motions. The governing equation of motions, have been presented in many works [1], [3], [4], [5], [6]. This model constitutes the basis for system analysis and control. The Euler or attitude angles, (Roll, Pitch, and Yaw) and the Cartesian coordinate are illustrated in figure 1, which represents the schematic of employed quadrotor. Two

- *Amir Nobahar Sadeghi Nam, Post-Doc. Research Fellow in Mechatronics engineering department, Atilim University, Ankara, Turkey*
  E-mail: amir.nobahar@gmail.com

interconnected subsystems presents the quadrotor dynamics. The first under-actuated subsystem gives the dynamics of the altitude with the Euler angles as the equations (1-4). The second fully-actuated subsystem gives the dynamics of the horizontal position, as the equations (5-6). In these equations, L is the distance between each rotor and center of the quadrotor, which places on the origin of the coordinate system. I_x , I_y , I_z are the inertias of the quadrotor in Cartesian coordinates. Also u_1, u_2, u_3, u_4 are roll, pitch, yaw and altitude control inputs respectively. As it is seen, the whole system is an under-ac y fc

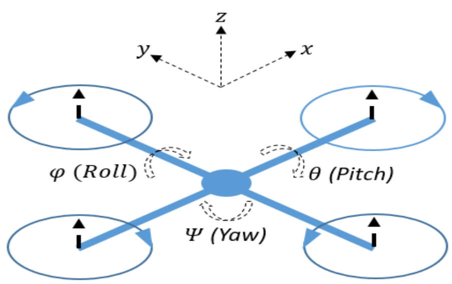


Fig. 1. Schematic of a Quadrotor with Euler Angles

$$\ddot{\varphi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\Psi} + \frac{L}{I_x}u_1 \quad (1)$$

$$\ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\varphi}\dot{\Psi} + \frac{L}{I_y}u_2 \quad (2)$$

$$\ddot{\Psi} = \frac{I_x - I_y}{I_z}\dot{\varphi}\dot{\theta} + \frac{L}{I_z}u_3 \quad (3)$$

$$\ddot{z} = -g + \frac{cos\varphi cos\theta}{m}u_4 \quad (4)$$

$$\ddot{x} = \frac{cos\varphi sin\theta cos\Psi + sin\varphi sin\Psi}{m}u_4 \quad (5)$$

$$\ddot{y} = \frac{cos\varphi sin\varphi sin\Psi - sin\varphi cos\Psi}{m}u_4 \quad (6)$$

3 Controller Design

In order to design motion or trajectory control system, two methodologies have been presented. In the first methodology, a classic PID controller is employed to achieve the desired trajectory, and in the second methodology, active disturbance rejection controller (ADRC) along with back stepping control architecture are designed to achieve the aim. It is worth mentioning that, the desired trajectory is traversed by changing of just only the roll and pitch angles of the quadrotors. In other words, the yaw angle is always zero, which means the quadrotor heading remains always parallel to the positive x-axis. The comparison of the designed controllers in the first and second methodologies is accomplished based on the performances measures presented in [11] as illustrated in the equations (7-10). In these equations, the error signal is the difference between the position reference input and measured position in x, y and z axes. The integral square error (ISE) represents the error energy, integral absolute error (IAE) determines the cumulative error, integral of time weighted absolute error (ITAE) displays the steady-state error, and the root mean square error (RMSE) represents the standard deviation of the errors.

$$ISE = \int e^2 dt \quad (7)$$

$$IAE = \int |e| dt \quad (8)$$

$$ITAE = \int t.|e| dt \quad (9)$$

$$RMSE = \int \frac{e^2}{n} dt \quad (10)$$

3.1 First Control Methodology

In the first methodology, employing a classic PID controller, the Euler angle outputs of the dynamic system is applied to the horizontal dynamics to track the given horizontal position references. The control schematic of this methodology is illustrated in figure 2. As mentioned, the yaw angle is always zero, hence the horizontal equation of motions will be as the equations (11-12). The horizontal position references, R_x and R_y, are tracked by changing roll and pitch angles, and the vertical position reference R_z is tracked by the produced vertical speed control input of the corresponding PID controller. These control inputs can be expressed as the equations (13-16).

$$\ddot{x} = \frac{cos\varphi sin\theta}{m}u_4 \quad (11)$$

$$\ddot{y} = -\frac{sin\varphi}{m}u_4 \quad (12)$$

$$u_1 = PID(R_x - x) \quad (13)$$

$$u_2 = PID\left(R_y - y\right) \quad (14)$$

$$u_3 = 0 \quad (15)$$

$$u_4 = PID(R_z - z) \quad (16)$$

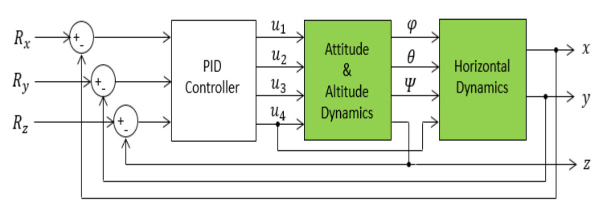


Fig. 2. Control Schematic based on the First Control Methodology

In the second methodology, back stepping control architecture is designed to achieve the desired trajectory. In addition, instead of a classic PID controller, an active disturbance rejection controller (ADRC) is employed to produce the corresponding control inputs. This type of controller is capable of eliminating the effect of all uncertain forces including the system parametric uncertainties and external disturbances. The control schematic of this methodology is illustrated in figure 3.

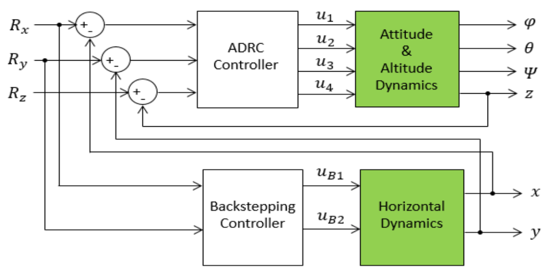


Fig. 3. Control Schematic based on the Second Control Methodology

3.2.1 Active Disturbance Rejection Controller Design

An active disturbance rejection controller (ADRC) consists of two main components: PD controller and Extended State Observer (ESO). This controller can successfully track the reference signal while rejecting all the parametric uncertainties and external disturbances. Schematic of ADRC controller for the horizontal and vertical motions dynamics is displayed in figure 4.

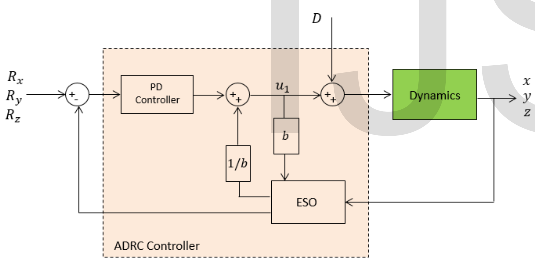


Fig. 4. Schematic of ADRC Controller

To design this observer for roll, pitch, yaw and altitude motions employing the equations (1-4), the dynamics of the observers should be extracted properly. Here the design procedure for roll dynamics is described, based on the equation (17). In this equation D_φ is the external disturbance effecting on the roll dynamics. The effect of the external disturbance and the model parametric uncertainties is considered as a new state "x_2". Equation (22) represents the observer dynamics. In the equation (23), "ω_0" is the observer bandwidth, which can be derived by bandwidth parameterization as presented in the reference [12].

$$\ddot{\varphi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\Psi} + \frac{L}{I_x}u_1 + D_\varphi \tag{17}$$

$$x_1 = \dot{\varphi}\ ,\ b_1 = \frac{L}{I_x}\ ,\ x_2 = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\Psi} \tag{18}$$

$$\dot{x}_1 = x_2 + b_1 u_1 \tag{19}$$

$$\dot{x}_2 = h \approx 0 \tag{20}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ h\end{bmatrix} + \begin{bmatrix}b_1\\ 0\end{bmatrix}\cdot u_1 \tag{21}$$

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = \begin{bmatrix}\hat{x}_2\\ 0\end{bmatrix} + \begin{bmatrix}b_1\\ 0\end{bmatrix}\cdot u + L_\varphi\cdot(x_1 - \hat{x}_1) \tag{22}$$

$$L_\varphi = [2\omega_0 \quad \omega_0^2] \tag{23}$$

The same procedure can be accomplished for pitch and altitude motions dynamics to design corresponding observers. But there is yet a problem in altitude dynamics presented in the equation (4). Since the coefficient of control input is a function of the states and not a constant value, so to solve this problem feedback linearization is applied. In other words we define the control input of altitude as below equation (24).

$$u_4 = (g - R_z)\frac{m}{cos\varphi cos\theta} \tag{24}$$

3.2.2 Back-Stepping Controller Design

Back stepping control is employed to stabilize the system. This technique is useful when some states are controlled through other states [13], [14]. The main objective is to design a controller ensuring that the horizontal position tracks the desired trajectory asymptotically. The design methodology is based on the *Lyapunov* stability theory.

In horizontal motion dynamics (equations 5-6), we have a system that can be written as $\dot{x} = A\cdot x + B_{(u)}\cdot u$. Our B matrix is a function of u, but what we need is A matrix, which is a constant matrix. As mentioned in reference book [14], a linear time invariant (LTI) system of the form $\dot{x} = A\cdot x$ can be considered in order to study the stability situation. Based on theorem 3.6 in this book, a necessary and sufficient condition for an LTI system to be strictly stable is that, for any symmetric positive definite matrix Q, the unique matrix P solution of the *Lyapunov* equation be symmetric positive definite. This theorem shows that any positive definite matrix Q can be used to determine the stability of a linear system. A simple choice of Q is the identity matrix. By considering a quadratic *Lyapunov* function candidate as $V = x^T Q x$, and differentiating the positive definite function V along the system trajectory yields another quadratic form as the equation (25). The corresponding P matrix in our horizontal dynamics is extracted presented in (26), which is positive definite, and therefore our position control system is globally asymptotically stable.

$$\dot{V} = \dot{x}^T Q x + x^T Q \dot{x} = x^T P x \quad ,\ P = A^T Q + QA \tag{25}$$

$$P = \begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix} \tag{26}$$

In order to design this controller for horizontal motion dynamics, and extract the corresponding control inputs, the below described procedure is implemented for x- axis dynamics.

$$\dot{x} = dx_1 = x_2 \tag{27}$$

$$dx_2 = \frac{cos\varphi sin\theta cos\Psi + sin\varphi sin\Psi}{m} u_4 \tag{28}$$

$$e_1 = R_x - x \rightarrow \dot{e_1} = \dot{R_x} - x_2 \tag{29}$$

If x_2 were the control input, by selecting $x_2 = c_1 e_1 + \dot{R_x}$ we will have $\dot{e_1} = -c_1 e_1$ and it guaranties exponential convergence of the error to zero. Here c_1 determines how fast the error converges to zero. Now, let consider the reference value for x_2 be as the equation (30), as if it were a virtual control input, where c_1 and λ_1 are positive constants.

$$R_{x_2} = c_1 e_1 + \dot{R_x} + \lambda_1 E_1 \quad , \quad E_1 = \int_0^t e_1(\tau)\, d\tau \tag{30}$$

$$e_2 = R_{x_2} - x_2 = c_1 e_1 + \dot{R_x} + \lambda_1 E_1 - x_2 \tag{31}$$

$$\rightarrow x_2 = c_1 e_1 + \dot{R_x} + \lambda_1 E_1 - e_2 \tag{32}$$

$$\dot{e_1} = \dot{R_x} - x_2 \rightarrow \dot{e_1} = -c_1 e_1 - \lambda_1 E_1 + e_2 \tag{33}$$

$$\dot{e_2} = \dot{R_{x_2}} - x_2 = c_1 \dot{e_1} + \ddot{R_x} + \lambda_1 e_1 - \dot{x_2} \tag{34}$$

$$\rightarrow \dot{e_2} = c_1(-c_1 e_1 - \lambda_1 E_1 + e_2) + \ddot{R_x} + \lambda_1 e_1 - \dot{x_2} \tag{35}$$

Let the desired dynamics for e_2 is given as $\dot{e_2} = -c_2 e_2 - e_1$, where c_2 is a positive constant, so we will have:

$$\dot{e_2} = -c_2 e_2 - e_1 = c_1(-c_1 e_1 - \lambda_1 E_1 + e_2) + \ddot{R_x} + \lambda_1 e_1 - \frac{cos\varphi sin\theta cos\Psi + sin\varphi sin\Psi}{m} u_4 \tag{36}$$

In horizontal position control system, because the outputs track R_x by changing just only in roll and pitch angles (yaw angle is always zero), so we can consider u_{B1} and u_{B2} as the control inputs of the horizontal dynamics as:

$$(1 + \lambda_1 - c_1{}^2)e_1 + (c_1 + c_2)e_2 - c_1\lambda_1 E_1 + \ddot{R_x} = \frac{cos u_{B1} \sin u_{B2}}{m} u_4 \tag{37}$$

Similarly it is extracted the following equation for y-axis dynamics:

$$(1 + \lambda_2 - c_3{}^2)e_3 + (c_3 + c_4)e_4 - c_3\lambda_2 E_2 + \ddot{R_y} = -\frac{sin\, u_{B1}}{m} u_4 \tag{38}$$

Then our needed control inputs are extracted as the below equations (39-40), based on the definitions presented in the equations (41-43). Here by tuning the positive parameters c_1 and c_2, the desired performance is expected. As mentioned, c_1 and c_2 determine how fast $x \rightarrow R_x$ and $y \rightarrow R_y$.

$$u_{B1} = \sin^{-1}\left(\frac{m((1 + \lambda_2 - c_3{}^2)e_3 + (c_3 + c_4)e_4 - c_3\lambda_2 E_2 + \ddot{R_y})}{u_4}\right) \tag{39}$$

$$u_{B2} = \sin^{-1}\left(\frac{-m((1 + \lambda_1 - c_1{}^2)e_1 + (c_1 + c_2)e_2 - c_1\lambda_1 E_1 + \ddot{R_x})}{u_4 \cos u_{B1}}\right) \tag{40}$$

$$e_1 = R_x - x_1 \quad , \quad e_2 = \dot{R_x} - x_2 \quad , \quad E_1 = \int_0^t e_1(\tau)\, d\tau \tag{41}$$

$$e_3 = R_y - x_3 \quad , \quad e_4 = \dot{R_y} - x_4 \quad , \quad E_2 = \int_0^t e_3(\tau)\, d\tau \tag{42}$$

$$c_2 = \frac{c_1{}^2 + 1}{c_1} \quad , \quad \lambda_1 = c_1{}^2 \quad , \quad c_4 = \frac{c_3{}^2 + 1}{c_3} \quad , \quad \lambda_2 = c_3{}^2 \tag{43}$$

4 Experiments and Results

To evaluate the behaviors of the controllers of the two presented control methodologies, the desired trajectory to track is considered as the follow path. Three swarm quadrotors place 3 meters away from each other on the corner of an equilateral triangle. That means they will be on the points at the Cartesian coordinates which is shown in figure 3. The quadrotors should start from the mentioned points and after traverse the designed trajectories come back again to these initial points.

(0,0,0) → (0,0,10) → (10,0,10) → (10,1,10) →
(0,10,10) → (0,0,10) → (0,0,0)

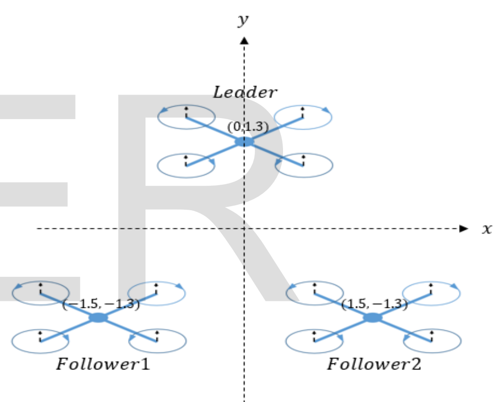


Fig. 3. Placement of Swarm Quadrotors

The behaviors of the controllers presented in first methodology is evaluated. In this control architecture, applying excessive external disturbance on the system, leads the controllers to go unstable. Figure 5, illustrate the position reference in three axes, measured attitude and position of the leader, figure 6, demonstrate the measured position in three axes of three swarm quadrotors, and figure 7, display the traversed trajectory by the swarm quadrotors employing the first methodology. As it is clearly seen, the measured horizontal and vertical positions can successfully track the given reference inputs.

Then the behaviors of the controllers presented in second methodology is evaluated. In this case by applying a disturbance signal on the attitude and altitude dynamics shown in the figure 8, the ADRC controller ability to eliminate it is studied. Unlike the applied disturbances in most of existing literature, the disturbance considered in this study is time varying and continuous. Figure 9, illustrate the position reference in three axes, measured attitude and position of the leader, figure 10, demonstrate the measured position in three axes of three swarm quadrotors, and figure 11, display the traversed trajectory by the swarm quadrotors employing the second methodology. Despite having disturbance on the dynamics, the measured horizontal and vertical positions can successfully track the given reference inputs.

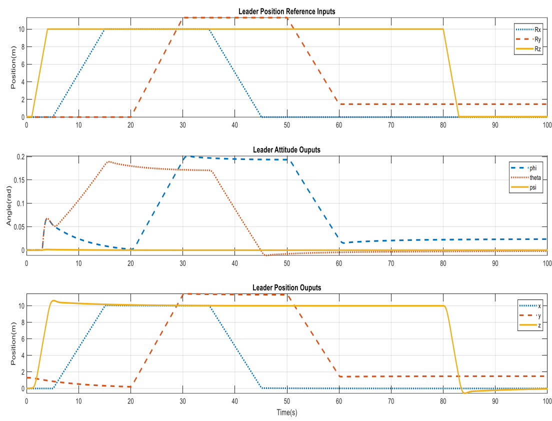


Fig. 5. Leader Position Reference, Attitude and Position Outputs, First Control Approach

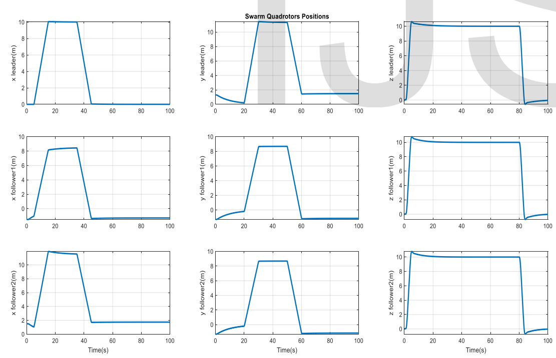


Fig. 6. Positions of Swarm Quadrotors, First Control Approach

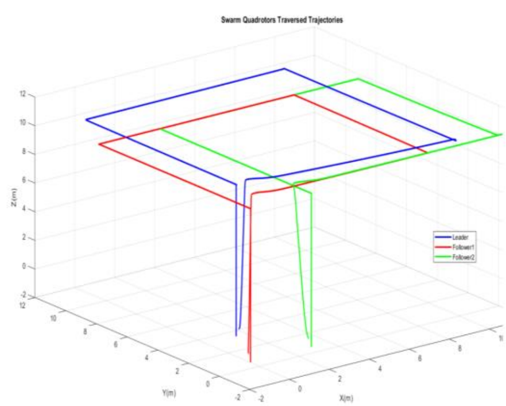
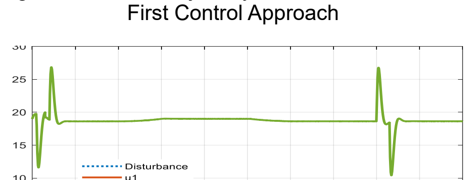


Fig. 7. Traversed trajectory of Swarm Quadrotors, First Control Approach



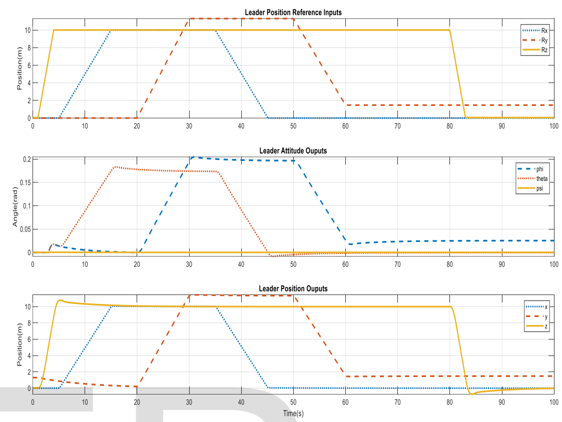


Fig. 9. Leader Position Reference, Attitude and Position Outputs, Second Control Approach

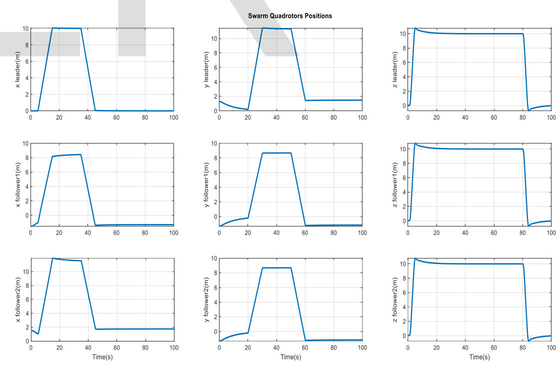


Fig. 10. Positions of Swarm Quadrotors, Second Control Approach

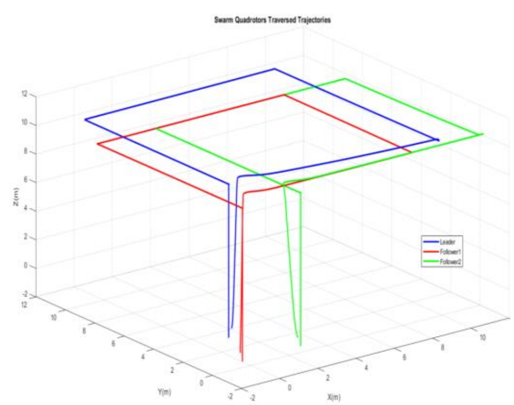


Fig. 7. Traversed trajectory of Swarm Quadrotors, Second Control Approach

In order to compare the behavior of the controllers in the first and second methodologies, the given reference position input, and measured positions in x, y and z axes of the leader quadrotor are extracted and presented in the figures 12-14. As a visual result, we can see in each methodology, the output signals can successfully track the input signals. To compare them more carefully, their performances measures are calculated as displayed in the table 1. Based on these values, we can conclude some notable issues. In compare with the first methodology, the second methodology has smaller error energy, gives the nearest response with respect to the applied reference, and has smaller steady-state error. Since in most of cases, the performance measures of the second methodology is smaller than the first methodology, it can be concluded that the back-stepping control along with an ADRC is more accurate in tracking the reference input of both horizontal and vertical dynamics. These preference is realized though annoying applied disturbances.

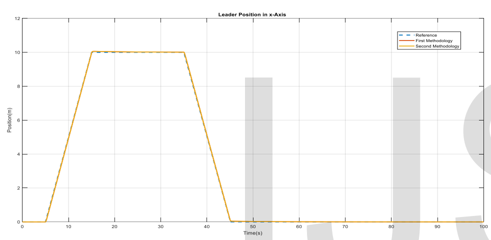


Fig. 12. Leader Position in x-axis, Reference Input, Outputs with First and Second Control Methodologies

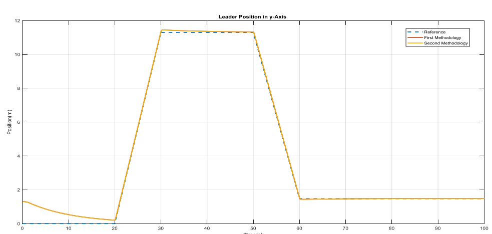


Fig. 13. Leader Position in y-axis, Reference Input, Outputs with First and Second Control Methodologies

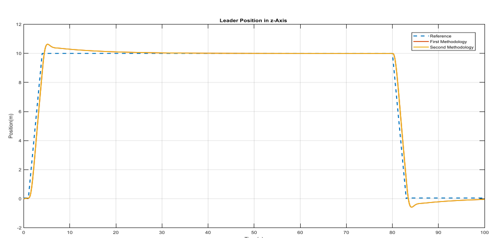


Fig. 14. Leader Position in z-axis, Reference Input, Outputs with First and Second Control Methodologies

5 Discussion and Conclusion

Control architectures comprising of classic PID controller, active disturbance rejection controller (ADRC) along with back stepping control architecture were designed and presented to drive the swarm quadrotors to a desired trajectory. Also a proper extended state observer (ESO) was designed to eliminate the external disturbances effecting on the system. The simulation results showed the good performance of both proposed control methodologies. In other words, the horizontal and vertical positions can successfully track the given reference inputs. The excellent performance of the second control methodology comes with elimination of the applied external disturbance on the attitude and altitude dynamics. The performance of both architectures were compared employing the mathematical control measures. It was concluded, the back-stepping control along with an ADRC is more accurate in tracking the desired reference input.

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